The co-evolutionary dynamics of directed network of spin market agents

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Abstract

The spin market model [S. Bornholdt, Int.J.Mod.Phys. C **12** (2001) 667] is extended into co-evolutionary version, where strategies of interacting and competitive traders are represented by local and global couplings between the nodes of dynamic directed stochastic network. The co-evolutionary principles are applied in the frame of Bak - Sneppen self-organized dynamics [P. Bak, K. Sneppen, Phys. Rev. Letter **71** (1993) 4083] that includes the processes of selection and extinction actuated by the local (node) fitness. The local fitness is related to orientation of spin agent with respect to instant magnetization. The stationary regime characterized by a fat tailed distribution of the log-price returns with index $\alpha \simeq 3.6$ (out of the Lévy range) is identified numerically. The non-trivial consequence of the extremal dynamics is the partially power-law decay (an effective exponent varies between -0.3 and -0.6) of the autocorrelation function of volatility. Broad-scale network topology with node degree distribution characterized by the exponent $\gamma = 1.8$ from the range of social networks is obtained.

Key words: spin market model, co-evolution, extremal dynamics, complex network
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The Ising model is the simplest model of the solid state that is an example of the N-body problem. From the perspective of economics this example has a great importance because it demonstrates that a basic interaction between the spins (agents) can bring a non-trivial collective phenomena. The parallels between the fluctuations in the economic and magnetic systems affords an application of the spin models to the market statistics. This motivation has been outlined by Cont and Buchaud in Ref.(1). There is suggested that the equilibrium distribution of the "super-spins" can be used to model the stock price fluctuations. The "super-spin" model has been extended by the Chowdhury

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and Stauffer Ref.(2) by switching on interactions between the lattice spins. Further works in this direction Refs.(3; 4; 5) have putted emphasis onto linkage of the notion of strategy (as a way of an agent's thinking) with the spin degrees of freedom. The combination of these particulars can be made on the basis of the famous concept of *minority game* Ref.(6). The spin variant of minority game assumes the competition of the ferromagnetic (short-range) and antiferromagnetic (global) couplings that cause an intermittent price dynamics of the stylised market. The salient points of the model Ref.(4) are reported in below.

The activity of *fundamentalists* and *interacting traders* has been imposed. Each interacting trader is characterized by a regular lattice site position i and by the corresponding spin variable $S^{(i)}(t)$ from $\{-1,1\}$. When $S^{(t)}(i)=1$, the agent tends to sell a unit amount of the stock whereas $S^{(t)}(i) = -1$ abbreviates the buy order during a given period. The variable $S^{(t)}(i)$ is updated by an asynchronous heat-bath dynamics

$$S^{(t+1)}(i) = \begin{cases} 1 & \text{with probability} \quad P(h^{(t)}(i)) \;, \\ -1 & \text{with probability} \quad 1 - P(h^{(t)}(i)) \;, \end{cases} \tag{1}$$

where $h^{(t)}(i)$ denotes the local field and $P(h)$ is the sigmoid function

$$P(h) = \frac{1}{1+\exp(-2\beta h)} \tag{2}$$

that depends on the inverse fictitious temperature β. The local field $h^{(t)}(i)$ reads

$$h^{(t)}(i) = J \sum_{j\in \mathrm{nn}(i)} S^{(t)}(j) - \kappa S^{(t)}(i)\,|\,m^{(t)}\,|\,. \tag{3}$$

Here $J>0$ is the exchange coupling of nearest neighbors $\mathrm{nn}(i)$ and $\kappa > 0$ is the coupling of the local field with instant magnetization

$$m^{(t)} = \frac{1}{L}\sum_{i=1}^{L} S^{(t)}(i)\,, \tag{4}$$

where L is the number of the sites at some lattice. When βJ is bellow the critical temperature, the term $\kappa S^{(t)}(i)|m^{(t)}|$ suppresses the ferromagnetic spin phase. For sufficiently large κ the frustration between the small (spin) and large (lattice) scales yields steady-state bubble-like regime. The determination of mutual orientation of spin and minority is only possible with use of integral

global information. Mathematically, the negative $S^{(t)}(i)m^{(t)}$ signalizes that agent i belongs to the minority. The integral information also affects the price of given stock written in the form

$$p^{(t)} = p^* \exp\left(\lambda m^{(t)}\right) , \tag{5}$$

where λ is the ratio of the number of fundamentalists to interacting traders. The formula can be interpteted as it follows: the predominance of buy orders implies $m^{(t)} > 0$ which causes that $p^{(t)}$ falls above the fundamental price p^*. Evidently, the negative $m^{(t)}$ corresponds to under-valued stock. Consequently, the change $m^{(t+1)} - m^{(t)}$ is associated with the logarithmic price return $\ln[p^{(t+1)}/p^{(t)}]$ in further.

1 Co-evolutionary dynamics on network of interacting spin agents

In order to enhance the realism of the model, we have suggested several modifications. The primarily inspiration for this originates from the evolutionary formulation of minority game that allows agents to adapt strategy according to past experience Ref.(7). As a result, the co-evolutionary spin market model is proposed that imposes the self-organized formation of the stationary distributions of *strategic variables* constituted by the set of *inherent fields* and *couplings.*

The static regular lattice geometry represents perhaps one of the most unrealistic aspects of aforementioned spin model. To overcome this problem in our approach we take advantage of directed network (graph) of labeled nodes $\Gamma = \{1, 2, \ldots, L\}$, where node $i \in \Gamma$ attaches via $N^{\rm out}$ directed links to its neighbors $X_n(i) \in \Gamma$, $n = 1, 2, \ldots, N^{\rm out}$, i.e. the graph is $N^{\rm out}$-regular. The links mediate non-symmetric spin-exchange couplings $J_n^{(t)}(i)$ through which the agents may exchange the game-relevant information. Two outgoing links $X_1(i) = 1 + (i) \mathrm{mod} L$, $X_2(i) = 1 + (L + i - 2) \mathrm{mod} L$ of node $i \in \Gamma$ are fixed to guarantee the network connectedness at any stage of its evolution. The dynamics of reconnections of the links $X_n(i)$, $n \geq 3$ (defined below) then develops on the background of the quenched bidirectional loop ($n = 1, 2$). In addition, the dynamics is constrained to forbid the self-connections ($X_n(i) = i$) as well as the multiple connections $[X_{n_1}(i) = X_{n_2 \neq n_1}(i)]$. An important information about the network statistics can be obtained by sampling the distribution of the node degrees. For our purposes here, we redefine *node degree* $k^{(\rm in)}(j) = \sum_{i=1}^{L} \sum_{n=1}^{N^{\rm out}} \delta_{j, X_n(i)}$ as a number that accounts exceptionally for the incoming links of the node j.

With the above topology in mind, the spin interaction is introduced via mod-

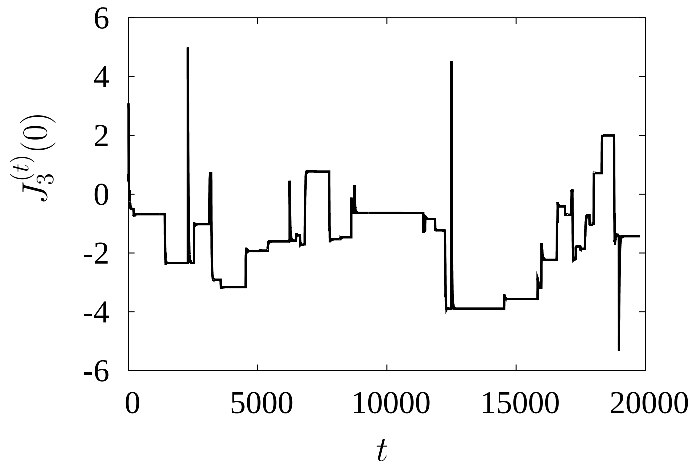

Fig. 1. The varying coupling $J_3^{(t)}(0)$ shows punctuated equilibrium behaviour, i.e., periods of peaceful epochs interrupted by an extremely short fluctuating periods of boots and reconnections which are the hallmarks of the self-organized criticality. We inspected that the probability density function (pdf) of J's indicates the formation of the "amorphous" and "glassy" mixture of the couplings that consists of ferromagnetic ($J_n^{(t)}(i) > 0$) and (prevailing) antiferromagnetic ($J_n^{(t)}(i) < 0$) contributions. The stationary mean value of $J_n^{(t)}(i)$ equal to -0.518 indicates the predominance of antiferromagnetic phase.

ified local field

$$h^{(t)}(i) = \frac{1}{N^{\rm out}} \sum_{n=1}^{N^{\rm out}} J_n^{(t)}(i)\, S^{(t)}\left(X_n^{(t)}(i)\right) + h_0^{(t)}(i) + \tilde{\kappa}^{(t)}(i)\, m^{(t)}\,. \tag{6}$$

We see that the strategic variable $\tilde{\kappa}^{(t)}(i)$ is reminiscent to the previously introduced global term. The inclusion of the node bias $h_0^{(t)}(i)$ closely resembles a random field model. It should be noted that preliminary simulations have confirmed qualitatively that the set of generated distributions is irrespective to variations in functional form of the local field.

As usual, the principles of the co-evolution are simply implementable in the frame of the local fitness as a phenotype. Here, the recurrently defined local fitness $F^{(t)}(i)$ is considered as an integrated history of the agent's spin orientations with respect to the minority

$$F^{(t+1)}(i) = F^{(t)}(i) - S^{(t)}(i)\, m^{(t)}\,. \tag{7}$$

The idea of the slowly (strategic) changing parameters Ref.(8) as a background of the spin dynamics is not a new within the family of spin market models. In order to optimize the local fitness, the co-evolutionary dynamics including the Hebbian and anti-Hebbian learning rule, respectively, has been proposed in Ref.(9). The approach proposed here follows similar lines, however, the

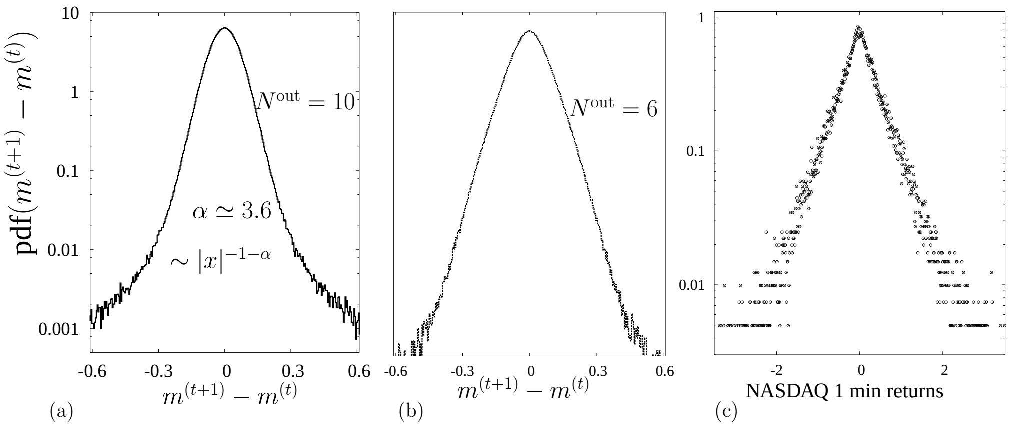

Fig. 2. The pdf of the log-price returns. In part (a) the index $\alpha = 3.6$ of $N^{\rm out} = 10$ links is extracted from $|x|^{-\alpha-1}$ fit. The central part of pdf is Gaussian distributed. Nearly exponential pdf in part (b) is obtained for $N^{\rm out} = 6$. We see that pdf attained by pruning of links resembles qualitatively Nasdaq composite (c) of 1 min data through May - September 2005.

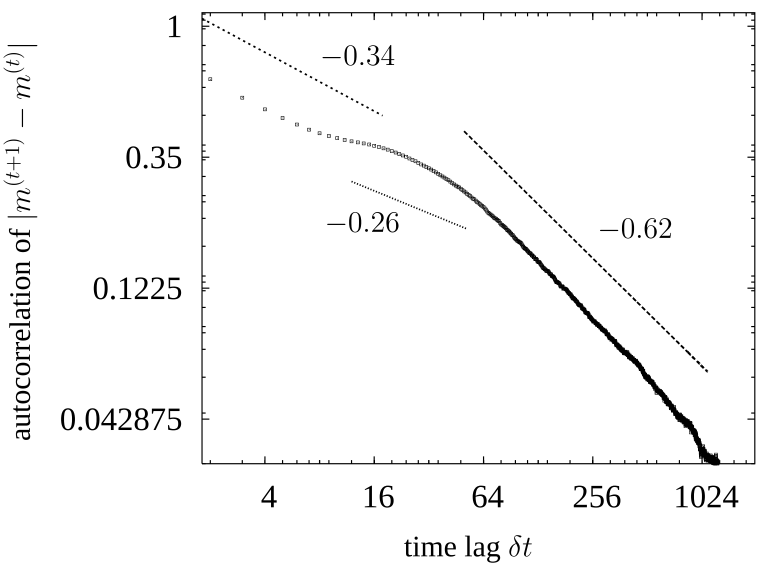

Fig. 3. The autocorrelation function of the volatility of the log-price returns $|m^{(t+1)} - m^{(t)}|$. The long-time memory with the particular effective decay exponents is identified numerically.

principal distinctions exist that are summarized in below:

(a) The strategies are subordinated to Bak - Sneppen extremal dynamics Refs.(10; 11; 12);

(b) In the present formulation we have $-S^{(t)}m^{(t)}$ instead of $-[\,S^{(t+1)}(i) - S^{(t)}(i)\,]m^{(t)}$ suggested in Ref.(9). The effect of the term involving the spin difference is that agent benefits from majority/minority jumps, while the abidance in minority is paradoxically penalized.

(c) The dynamical network of spin couplings is permanently rewired which mimics the social effects, where emphasis is putted on the links to the fittest

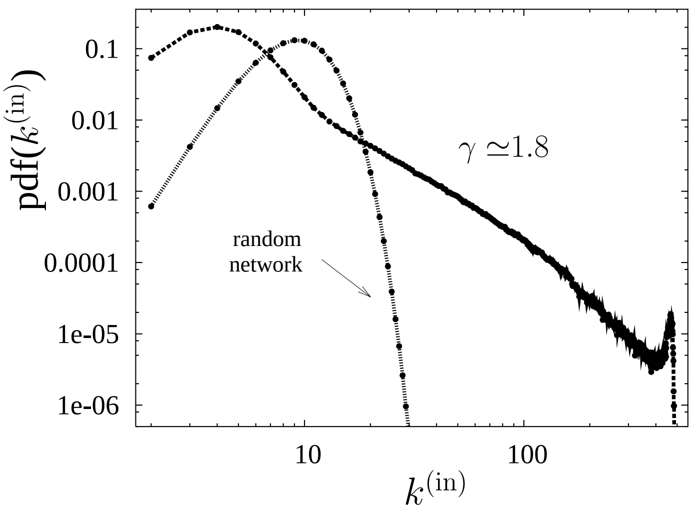

Fig. 4. The pdf's of the network node degree $k^{(\mathrm{in})}(j)$ calculated for $L = 500$, $N^{\mathrm{out}} = 10$. The comparison with the poissonian distribution obtained for the situation when the followers attach to randomly chosen node (no matter how large is the fitness). For the attachments to leader and its neighbors imposed the $[k^{(\mathrm{in})}]^{-\gamma}$ tail is formed. The exponent $\gamma \sim 1.8$ corresponds to the fit within the range $< 13, 50 >$.

nodes. Generally, our approach resembles the concept of preferential attachments that comes from the core of the proposal of the Barabási and Albert Ref.(13).

Recently, the interest in complex networks has been extended to the search for local rules governing to dynamics of social networks. Several principles have been exploited for this purpose. As an example let us mention the network transformation due to node aging and time-dependent attachment probability Ref.(14). The model proposed here is however closer to paradigm of inter-agent communication across net Refs.(15; 16).

We have considered slow motion of *strategic variables* defined by the recursive cycles over the following steps:

(1) The information that leader agent labeled as $i_{\max}$ has achieved the highest fitness $F(i_{\max}) \geq F(j)$, $\forall j \in \Gamma$ is available to all agents. Clearly, the idealized and unbounded access to information resources monitoring the instant fitness of all agents is assumed.
(2) The assumption of the model is that the strategic transfer from *leader* to the crowd of $L-1$ followers $i \in \Gamma$, $i \neq i_{\max}$ is carried out independently of the local network structure. The supposition about the collective action of the crowd expresses the *bounded rationality* of the *followers*. In essence, all the followers believe that imitation of strategy that has been applied in the aforegoing cycle (i.e. after 1 visit per node) by the most successful trader $i_{\max}$ would bring a future benefit to them. We postulated that each follower i adapts the strategy of leader according to prescription

$$
\begin{aligned}
J_n^{(\mathrm{next})}(i) &= J_n(i)\,(\,1-\eta_{J,\,n}(i)\,) + J_n(i_{\max})\eta_{J,\,n}(i)\,, && (8)\\
h_0^{(\mathrm{next})}(i) &= h_0(i)\,(1-\eta_{h_0}(i)\,) + h_0(i_{\max})\eta_{h_0}(i)\,, && (9)\\
\tilde{\kappa}^{(\mathrm{next})}(i) &= \tilde{\kappa}(i)\,(\,1-\eta_{\tilde{\kappa}}(i)\,) + \tilde{\kappa}(i_{\max})\eta_{\tilde{\kappa}}(i)\,, && (10)\\
P_{\mathrm{lead}}^{(\mathrm{next})}(i) &= P_{\mathrm{lead}}(i)\,(\,1-\eta_{PL}(i)\,) + P_{\mathrm{lead}}(i_{\max})\eta_{PL}(i)\,, && (11)\\
P_{\mathrm{T}}^{(\mathrm{next})}(i) &= P_{\mathrm{T}}(i)\,(\,1-\eta_{PT}(i)\,) + P_{\mathrm{T}}(i_{\max})\eta_{PT}(i)\,, && (12)
\end{aligned}
$$

The formula expresses the individual adaption differences. They are reflected by the plasticity parameters $\eta_{J,\,n}(i)$, $\eta_{h_0}(i)$, $\eta_{\tilde{\kappa}}(i)$ $\eta_{PT}(i)$, $\eta_{PL}(i)$ uniformly distributed over the range $(0, \eta_{\max})$, where $0 < \eta_{\max} < 1$.

(3) What have been left in the previous step without explanation are local probabilities of preferential attachments $P_{\mathrm{lead}}(i)$, $P_{\mathrm{T}}(i)$. They occur in the specific local rules of network dynamics. For given i and randomly selected $n, n_1 \geq 3$ the rewiring follows one of three probabilistic channels

$$
\begin{array}{lll}
X_n(i) & \left[\begin{array}{l}\text{is left}\\ \text{without change}\end{array}\right] & \text{with probability } 1 - P_{\mathrm{lead}}(i)\,,\\
X_n(i) & \leftarrow i_{\max} & \text{with probability } (1-P_{\mathrm{T}}(i))\,P_{\mathrm{lead}}(i)\,,\\
X_{n_1\neq n}(i) & \leftarrow X_{n_1}(i_{\max}) & \text{with probability } P_{\mathrm{lead}}(i)\,P_{\mathrm{T}}(i)\,,
\end{array}
\qquad (13)
$$

where $(1-P_{\mathrm{T}}(i))P_{\mathrm{lead}}(i)$ is the probability of the connection to node of actual leader. The update $X_n(i) \leftarrow X_{n_1}(i_{\max})$ favors the *social transitivity* Ref.(17). In addition, the transitivity requires bounding $P_{\mathrm{T}}(i) \leq \epsilon_{\mathrm{T}} \sim O(1) \gg P_{\mathrm{lead}}$.

(4) At given t the synchronous update $J_n(i) \leftarrow J_n^{(\mathrm{next})}(i)$, $\tilde{\kappa}(i) \leftarrow \tilde{\kappa}^{(\mathrm{next})}(i)$, $h_0(i) \leftarrow h_0^{(\mathrm{next})}(i)$, $P_{\mathrm{lead}}(i) \leftarrow P_{\mathrm{lead}}^{(\mathrm{next})}(i)$, $P_{\mathrm{T}}(i) \leftarrow P_{\mathrm{T}}^{(\mathrm{next})}(i)$ is carried out for all $i \in \Gamma$ and $n = 1, 2, \ldots, N^{\mathrm{out}}$.

(5) The agent $i_{\min}$ of the lowest fitness $F(i_{\min})$ finishes her/his unsuccessful life via *extremal dynamics*. The death-birth rules $J_n^{(t+1)}(i_{\min}^{(t)}) = (2r_{1,n}^{(t)} - 1)\epsilon_{\mathrm{J}}$, $h_0^{(t+1)}(i_{\min}^{(t)}) = (2r_3^{(t)} - 1)\epsilon_{\mathrm{h_0}}$, $\tilde{\kappa}^{(t+1)}(i_{\min}^{(t)}) = (2r_4^{(t)} - 1)\epsilon_{\kappa}$ with integer $X_n^{(t+1)}(i_{\min}^{(t)}) = [1 + Lr_{2,n}^{(t)}]_{\mathrm{int}}$ and strictly positive $P_{\mathrm{lead}}^{(t+1)}(i_{\min}^{(t)}) = r_5\epsilon_{\mathrm{lead}}$, $P_{\mathrm{T}}^{(t+1)}(i_{\min}^{(t)}) = r_6^{(t)}\epsilon_{\mathrm{T}}$ are driven by the uncorrelated uniformly distributed numbers $\{r_{1,n}^{(t)}\}_{n=1}^{N^{\mathrm{out}}}$, $\{r_{2,n}^{(t)}\}_{n=1}^{N^{\mathrm{out}}}$, $\{r_j^{(t)}\}_{j=3}^{7}$ from $(0,1)$. The last $r_7^{(t)}$ serves for "nearly middle" setting $F^{(t+1)}(i_{\min}^{(t)}) = F^{(t)}(i_{\min}^{(t)}) + r_7^{(t)}(F(i_{\max}^{(t)}) - F(i_{\min}^{(t)}))$. The random numbers depend on the free parameters $\epsilon_{\mathrm{J}}, \epsilon_{\mathrm{h0}}, \epsilon_{\kappa}, \epsilon_{\mathrm{lead}}, \epsilon_{\mathrm{T}}$ that bound the available strategic space. The same updates have been used for imposition of initial conditions. As one can see from the structure of updates, the adaptivity in common with extremal dynamics preserves the span of the strategic space.

2 Numerical results

The simulation has been carried out for the spin system thermalized via the asynchronous Glauber dynamics with $\beta = 1$ within the strategic bounds $\epsilon_{\rm lead} = 0.01$, $\epsilon_{\rm T} = 0.5$, $\epsilon_{\rm J} = 8$, $\epsilon_{\kappa} = \epsilon_{\rm h_0} = 4$, and with topology $L = 500$, $N^{\rm out} = 10$ for $t \leq 2 \times 10^7$ iterations initialized from random initial settings [see step (5) of the algorithm]. As usual, the data corresponding to transient regime has been discarded. Hereafter we summarize the main findings of simulation.

The self-organization process of intermittent fluctuations of the log-price returns has been obtained by the simulation. The punctuated equilibrium of selected strategic variable is monitored in Fig 1.

The relevance of the approach for the economic modeling is demonstrated by the fat tail probability density function (pdf) of the log-price returns Ref.(18). It can be roughly characterized by the exponent $\alpha \simeq 3.6$ (Fig.2). At this moment, it is rather illustrative to mention the empirical value $\alpha = 3.1$ corresponding to 1-day period of TAQ database Ref.(19). It has been checked numerically that distribution may vary with the bounding of strategic space and chosen topology ($N^{\rm out}$). The generalization proposed herein covers not only power-law but also nearly exponential or Gaussian-like distributions.

In Fig.3 the emergence of the long market memory is demonstrated by calculating the autocorrelation of volatility of the log-price returns. The empirical slopes -0.34 Ref.(19) and -0.2 Ref.(20) should be mentioned in this context. As it turns out from our simulation, the co-evolution represents one of the potential mechanisms responsible for the power-law decay. More specifically, we have shown the inclusion of *extremal dynamics and adaptivity* can cure the notorious exponential decay problem of the spin market models. Even more interestingly, only the *simultaneous exploiting of both mentioned dynamical principles can result in nontrivial autocorrelations.* Within the spin market models, an alternative explanation to our has been proposed only recently Ref.(5). In the quoted work, the power-law decay is kept for three-state Potts model where zero spin state encodes the inactive (waiting) state of the trader.

In the stationary regime the self-organized reconstruction yields network of the broad-scale distributed degrees of nodes (see Fig.4). For the upper part of the power-law range we have estimated an exponent $\gamma \simeq 1.8$. The result is quite encouraging since the exponent falls within the range typical for the social networks. Here we report on a comparison the exponents of several cooperative social networks: the value $\gamma = 1.81$ Ref. (21) is obtained by collecting the e-mal addresses, the exponent 1.2 belongs to the coauthorship

network Ref. (22) whereas 2.1 characterizes the phone-call network Ref. (23). Similarly as in Ref.(24), the network topology can be characterized by the clustering coefficient C_i or its spatial-temporal mean $\langle C \rangle$. For directed network we have used slightly modified formula $C_i = e_i/(N^{\text{out}}(1 - N^{\text{out}}))$, where $e_i = \sum_{n_1,n_2,n_3=1}^{N^{\text{out}} \times N^{\text{out}} \times N^{\text{out}}} \delta_{X_{n_1}(X_{n_2}(i)),X_{n_3}(i)}$ stands for the number of links between the neighbors of some node i. In this formula, $N^{\text{out}}\,(1 - N^{\text{out}})$ represents the maximum number of links going from $X_{n_2}(i)$ to $X_{n_3}(i)$ and vice versa. It should be noted that the conventional factor 2 absents in the definition since the links are directed. As usual, it is meaningful to compare the mean clustering coefficients of two simulated network regimes. For randomly attached nodes [instead of selection declared by Eq.(13)] the simulation results in $\langle C_{\text{rand}} \rangle \simeq 0.027$, while $\langle C \rangle \simeq 0.125$ stems from the preference of $i_{\max}$, and thus the enhancement of clustering is confirmed by the ratio $\langle C \rangle / \langle C_{\text{rand}} \rangle \simeq 4.6$.

3 Conclusions

The generalized version of the spin market model has been suggested and investigated numerically. In the approach we used the static couplings are adjusted by an instant local fitness that drives the co-evolutionary competitive changes. The socially relevant feature of the approach is slowly evolving topology of the spin-spin interaction network as a substrate for fast buy-sell spin decisions. The power-law distributions have been uncovered as an attributes of the universality caused by the extremal dynamics. Our results suggest hypothesis that slowly varying strategic variables can cause an emergence of the long memory effect that is symptomatic for the real markets. The universality manifests itself in a broad-scale node degree distribution, however, *the network evolution does not necessary imply the power-law decay of volatility*. Even the static net enough to gain the power-law decay or flat-tailed pdf's. Numerous examples of the model applicability can be found outside the field of econophysics. For instance, the coevolutionary model opens the possibility of the optimization of the spin lattice and related systems in the manner of Ref.(25). The finite-size scaling analysis, the small world behaviour and the aspect of modularity are only a few examples of issues that remain to be analyzed in the complementary studies.

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